Name: _____ **Class:** _____ **Date:** _____ **ID: A**

MAC 2311 Review 2.6 - 2.9

Numeric Response

1. Calculate y'.

   $xy^4 + x^2y = 2x + 3y$

2. Calculate y'.

   $\cos(xy) = x^6 - y$

3. The position function of a particle is given by

   $s = t^3 - 10.5t^2 - 2t$, $t \geq 0$

   When does the particle reach a velocity of 22 m/s?

4. A spherical balloon is being inflated. Find the rate of increase of the surface area $S = 4\pi r^2$ with respect to the radius r when $r = 1$ ft.

5. Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is $F = \dfrac{GmM}{r^2}$.

   Find $\dfrac{dF}{dr}(7)$.

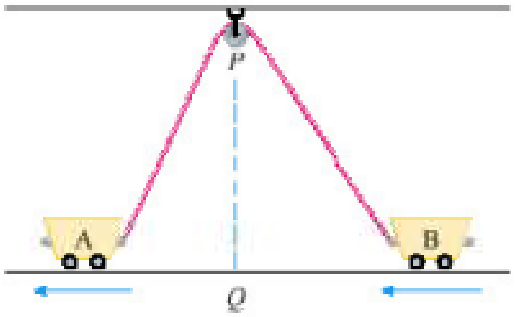
6. The position function of a particle is given by

   $s = t^3 - 10.5t^2 - 2t$, $t \leq 0$

   When does the particle reach a velocity of 22 m/s?

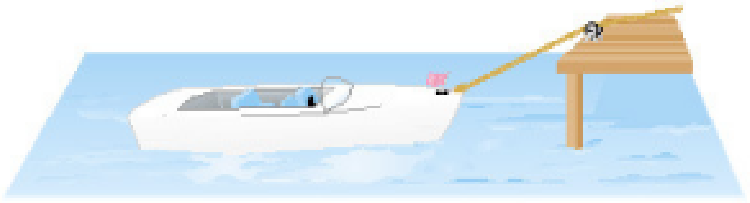
Name: _____ **ID: A**

7. Two carts, A and B, are connected by a rope 39 ft long that passes over a pulley (see the figure below). The point Q is on the floor 12 ft directly beneath and between the carts. Cart A is being pulled away from *Q* at a speed of 4 ft/s. How fast is cart B moving toward Q at the instant when cart A is 5 ft from Q?



8. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of $2\ \text{cm}^2/\text{min}$. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is $100\ \text{cm}^2$.

9. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s how fast is the boat approaching the dock when it is 6 m from the dock? Round the result to the nearest hundredth if necessary.



10. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 40 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base? Round the result to the nearest hundredth.

Short Answer

1. Find the tangent line to the ellipse $\frac{x^2}{40} + \frac{y^2}{10} = 1$ at the point $\left(2,\ -\sqrt{3}\right)$.

2. Find $\frac{dy}{dx}$ by implicit differentiation.

   $9x^2 - 8y^2 = 9$

3. Use implicit differentiation to find an equation of the tangent line to the curve at the indicated point.

   $y = \sin xy^6; \quad \left(\frac{\pi}{2}, 1\right)$

4. Calculate y'.

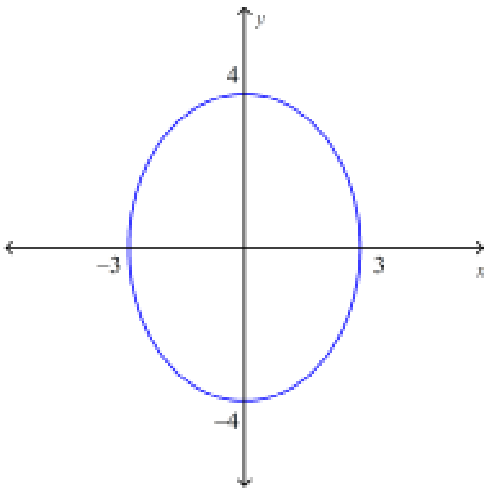
   $xy^3 + x^3y = x + 3y$

5. Find the derivative of the function.

   $y = 3\cos^{-1}\left(\sin^{-1} t\right)$

6. Find an equation of the tangent line to the given curve at the indicated point.

   $\frac{x^2}{9} + \frac{y^2}{16} = 1; \quad \left(2, \frac{4\sqrt{5}}{3}\right)$



7. Find $\frac{dy}{dx}$ by implicit differentiation.

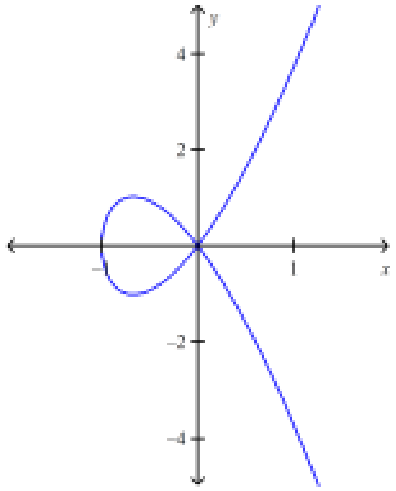
   $8\sqrt{x} + \sqrt{y} = 8$

8. Find an equation of the tangent line to the curve

   $xe^{y} + x + 2y = 2$ at (1, 0).

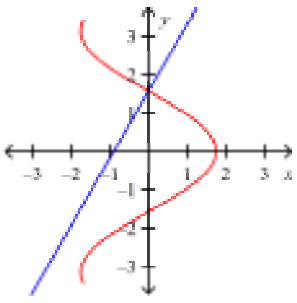
9. Find an equation of the tangent line to the given curve at the indicated point.

   $\frac{1}{7}y^2 - x^3 - x^2 = 0; \quad \left(1, \sqrt{14}\right)$



10. Two curves are said to be **orthogonal** if their tangent lines are perpendicular at each point of intersection of the curves. Show that the curves of the given equations are orthogonal.

    $y - \frac{7}{4}x = \frac{\pi}{2}, \quad x = \frac{7}{4}\cos y$



11. The equation of motion is given for a particle, where s is in meters and t is in seconds. Find the acceleration after 2.5 seconds.

    $s = \sin 2\pi t$

12. The mass of the part of a metal rod that lies between its left end and a point x meters to the right is

    $S = 4x^2$.

    Find the linear density when x is 3 m.

13. Find the rate of change of y with respect of x at the indicated value of x.

    $t = \csc x - 18 \cos x; \quad x = \dfrac{\pi}{6}$

14. In an adiabatic process (one in which no heat transfer takes place), the pressure P and volume V of an ideal gas such as oxygen satisfy the equation

    $P^5 V^7 = C$,

    where C is a constant. Suppose that at a certain instant of time, the volume of the gas is 2L, the pressure is 100 kPa, and the pressure is decreasing at the rate of 5 kPa/sec. Find the rate at which the volume is changing.

15. The mass of part of a wire is $x\left(1 + \sqrt{x}\right)$ kilograms, where x is measured in meters from one end of the wire. Find the linear density of the wire when $x = 36m$.

16. If f is the focal length of a convex lens and an object is placed at a distance v from the lens, then its image will be at a distance u from the lens, where f, v, and u are related by the *lens equation*

    $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u}$.

    Find the rate of change of v with respect to u.

17. Suppose that f and g are functions that are differentiable at $x = 2$ and that $f(2) = 1, f'(2) = -5$, $g(2) = -3$, and $g'(2) = -2$. Find $h'(2)$.

    $h(x) = \dfrac{xf(x)}{x + g(x)}$

18. $s(t)$ is the position of a body moving along a coordinate line; $s(t)$ is measured in feet and t in seconds, where $t \geq 0$. Find the position, velocity, and speed of the body at the indicated time.

    $s(t) = t^{10}e^{-t}; \qquad t = 1$

19. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the polluted area is circular, determine how fast the area is increasing when the radius of the circle is 20 ft and is increasing at the rate of $\frac{1}{6}$ ft/sec. Round to the nearest tenth if necessary.

20. Two chemicals react to form another chemical. Suppose that the amount of chemical formed in time t (in hours) is given by

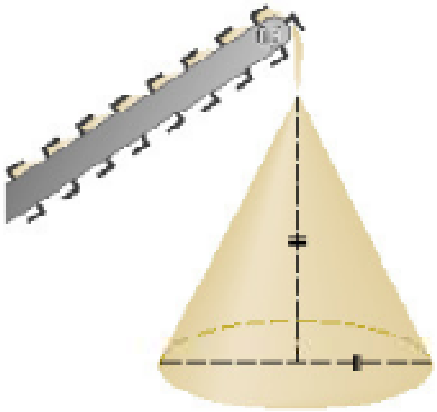
$$x(t) = \frac{11\left[1 - \left(\frac{2}{3}\right)^{3t}\right]}{1 - \frac{1}{4}\left(\frac{2}{3}\right)^{3t}}$$

    where $x(t)$ is measured in pounds.
    a. Find the rate at which the chemical is formed when $t = 4$. Round to two decimal places.
    b. How many pounds of the chemical are formed eventually?

21. Two cars start moving from the same point. One travels south at 70 mi/h and the other travels west at 20 mi/h. At what rate is the distance between the cars increasing 2 hours later? Round the result to the nearest hundredth.

22. Two sides of a triangle are 2 m and 3 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.

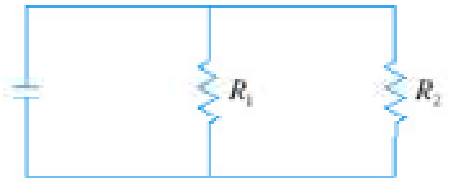
23. Gravel is being dumped from a conveyor belt at a rate of 34 ft/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 13 ft high? Round the result to the nearest hundredth.



24. If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R measured in ohms (Ω), is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of $0.1\ \Omega/s$ and $0.3\ \Omega/s$ respectively, how fast is R changing when $R_1 = 75$ and $R_2 = 100$?
Round the result to the nearest thousandth.



25. A point moves along the curve $3y + y^2 - 8x = 2$. When the point is at $\left(-\frac{1}{2}, -1\right)$, its x-coordinate is increasing at the rate of 3 units per second. How fast is its y-coordinate changing at that instant of time?

26. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s . At the moment when the bottom of the ladder is 1.5 m from the wall, it slides away from the wall at a rate of 0.3 m/s. How long is the ladder?

27. The top of a ladder leaning against a wall is 8 ft above the ground. The slope of the ladder with respect to the ground is –4. What is the length of the ladder?

28. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the polluted area is circular, determine how fast the area is increasing when the radius of the circle is 20 ft and is increasing at the rate of $\frac{1}{6}$ ft/sec. Round to the nearest tenth if necessary.

MAC 2311 Review 2.6 - 2.9
Answer Section

NUMERIC RESPONSE

1. ANS: $y' = \dfrac{2 - y^4 - 2xy}{4xy^3 + x^2 - 3}$

   PTS: 1 DIF: Medium REF: 2.6.23 KEY: 7e
   MSC: Numerical Response NOT: Section 2.6

2. ANS: $y' = \dfrac{6x^5 + y\left(\sin\left(xy\right)\right)}{1 - x\left(\sin\left(xy\right)\right)}$

   PTS: 1 DIF: Medium REF: 3.5.17 KEY: 7e
   MSC: Numerical Response NOT: Section 3.5

3. ANS: 8

   PTS: 1 DIF: Medium REF: 2.7.9a KEY: 7e
   MSC: Numerical Response NOT: Section 2.7

4. ANS: 8π

   PTS: 1 DIF: Medium REF: 2.7.15 KEY: 7e
   MSC: Numerical Response NOT: Section 2.7

5. ANS: $\dfrac{-2GmM}{343}$

   PTS: 1 DIF: Medium REF: 2.7.20a KEY: 7e
   MSC: Numerical Response NOT: Section 2.7

6. ANS: 8

   PTS: 1 DIF: Medium REF: 2.7.10a KEY: 7e
   MSC: Numerical Response NOT: Section 2.7

7. ANS: 1.73 ft/s

   PTS: 1 DIF: Medium REF: 2.8.38 KEY: 7e
   MSC: Numerical Response NOT: Section 2.8

8. ANS: -1.6

   PTS: 1 DIF: Medium REF: 2.8.19 KEY: 7e
   MSC: Numerical Response NOT: Section 2.8

9. ANS: 1.01

PTS: 1 DIF: Medium REF: 2.8.20 KEY: 7e
MSC: Numerical Response NOT: Section 2.8

10. ANS: 17.89 ft/s

PTS: 1 DIF: Medium REF: 2.8.18a KEY: 7e
MSC: Numerical Response NOT: Section 2.8

SHORT ANSWER

1. ANS:
$y = \frac{\sqrt{3}}{6}x - \frac{4\sqrt{3}}{3}$

PTS: 1 DIF: Medium REF: 3.5.76 KEY: 7e
MSC: Bimodal NOT: Section 3.5

2. ANS:
$\frac{9x}{8y}$

PTS: 1 DIF: Medium REF: 2.6.1 KEY: 7e
MSC: Bimodal NOT: Section 2.6

3. ANS:
$y = 1$

PTS: 1 DIF: Difficult REF: 2.6.26 KEY: 7e
MSC: Bimodal NOT: Section 2.6

4. ANS:
$y' = \frac{1 - y^3 - 3x^2y}{3xy^2 + x^3 - 3}$

PTS: 1 DIF: Medium REF: 2.6.8 KEY: 7e
MSC: Bimodal NOT: Section 2.6

5. ANS:
$y' = -\frac{3}{\sqrt{\left(1 - t^2\right)\left(1 - \left(\sin^{-1}(t)\right)^2\right)}}$

PTS: 1 DIF: Medium REF: 3.5.58 KEY: 7e
MSC: Bimodal NOT: Section 3.5

6. ANS:
$y = -\frac{8\sqrt{5}}{15}x + \frac{12\sqrt{5}}{5}$

PTS: 1 DIF: Medium REF: 2.6.27 KEY: 7e
MSC: Bimodal NOT: Section 2.6

7. ANS:
$-\frac{8\sqrt{y}}{\sqrt{x}}$

PTS: 1 DIF: Medium REF: 2.6.6 KEY: 7e
MSC: Short Answer NOT: Section 2.6

8. ANS:
$y = -\frac{2}{3}x + \frac{2}{3}$

PTS: 1 DIF: Medium REF: 3.5.45 KEY: 7e
MSC: Short Answer NOT: Section 3.5

9. ANS:
$y = \frac{5\sqrt{14}}{4}x - \frac{\sqrt{14}}{4}$

PTS: 1 DIF: Difficult REF: 2.6.29 KEY: 7e
MSC: Short Answer NOT: Section 2.6

10. ANS:
The curves intersect at $\left(0, \frac{\pi}{2}\right)$.
For $y - \frac{7}{4}x = \frac{\pi}{2}$, $m = \frac{7}{4}$.
For $x = \frac{7}{4}\cos y$, $m = -\frac{4}{7}\csc y$; at $\left(0, \frac{\pi}{2}\right)$, $m = -\frac{4}{7}$.

PTS: 1 DIF: Medium REF: 2.6.51 KEY: 7e
MSC: Short Answer NOT: Section 2.6

11. ANS:
0 m/s^2

PTS: 1 DIF: Medium REF: 2.7.3g KEY: 7e
MSC: Bimodal NOT: Section 2.7

12. ANS:
24

PTS: 1 DIF: Medium REF: 2.7.17a KEY: 7e
MSC: Bimodal NOT: Section 2.7

13. ANS:
$9 - 2\sqrt{3}$

PTS: 1 DIF: Medium REF: 2.7.16b KEY: 7e
MSC: Bimodal NOT: Section 2.7

14. ANS:
$\frac{1}{14}$ L/sec

PTS: 1 DIF: Medium REF: 2.7.23a KEY: 7e
MSC: Bimodal NOT: Section 2.7

15. ANS:
$9\,\text{kg}/\text{m}$

PTS: 1 DIF: Medium REF: 2.7.17a KEY: 7e
MSC: Bimodal NOT: Section 2.7

16. ANS:
$\frac{dv}{du} = -\frac{f^2}{\left(u-f\right)^2}$

PTS: 1 DIF: Medium REF: 2.7.33 KEY: 7e
MSC: Bimodal NOT: Section 2.7

17. ANS:
11

PTS: 1 DIF: Difficult REF: 2.7.33 KEY: 7e
MSC: Short Answer NOT: Section 2.7

18. ANS:
$\frac{1}{e}$ ft, $\frac{9}{e}$ ft/sec, $\frac{9}{e}$ ft/sec

PTS: 1 DIF: Medium REF: 3.7.4 KEY: 7e
MSC: Short Answer NOT: Section 3.7

19. ANS:
$20.9\ \text{ft}^2/\text{sec}$

PTS: 1 DIF: Medium REF: 2.7.19 KEY: 7e
MSC: Short Answer NOT: Section 2.7

20. ANS:
a. 0.08 lb/hr, b. 11 lbs

PTS: 1 DIF: Medium REF: 2.7.18a KEY: 7e
MSC: Short Answer NOT: Section 2.7

21. ANS:
72.80 mi/h

PTS: 1 DIF: Medium REF: 2.8.15 KEY: 7e
MSC: Bimodal NOT: Section 2.8

22. ANS:
$0.090\ m^2/s$

PTS: 1 DIF: Medium REF: 2.8.29 KEY: 7e
MSC: Bimodal NOT: Section 2.8

23. ANS:
0.26 ft/min

PTS: 1 DIF: Medium REF: 2.8.27 KEY: 7e
MSC: Bimodal NOT: Section 2.8

24. ANS:
$0.0878\ \Omega/s$

PTS: 1 DIF: Medium REF: 2.8.35 KEY: 7e
MSC: Bimodal NOT: Section 2.8

25. ANS:
24 units/sec

PTS: 1 DIF: Medium REF: 2.8.10 KEY: 7e
MSC: Bimodal NOT: Section 2.8

26. ANS:
3.4 m

PTS: 1 DIF: Medium REF: 2.8.31 KEY: 7e
MSC: Bimodal NOT: Section 2.8

27. ANS:
$2\sqrt{17}$ ft

PTS: 1 DIF: Medium REF: 2.8.31 KEY: 7e
MSC: Short Answer NOT: Section 2.8

28. ANS:
20.9 ft^2/sec

PTS: 1 DIF: Medium REF: 2.8.6 KEY: 7e
MSC: Short Answer NOT: Section 2.8